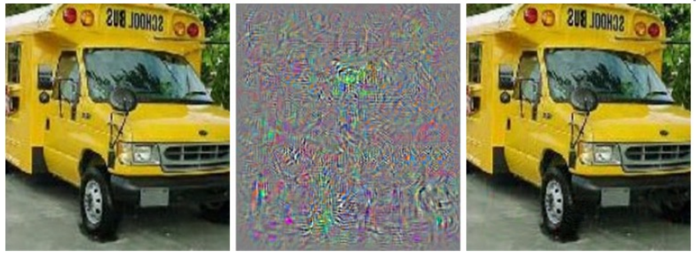
DEEL
DEpendable & Explainable Learning

1-LIPSCHITZ NEURAL NETWORKS

Mathieu Serrurier, Franck Mamalet, Alberto Gonźalez-Sanz, Thibaut Boissin, Jean-Michel Loubes, Eustasio del Barrio

CONTEXT

- Adversarial attacks of Neural Networks

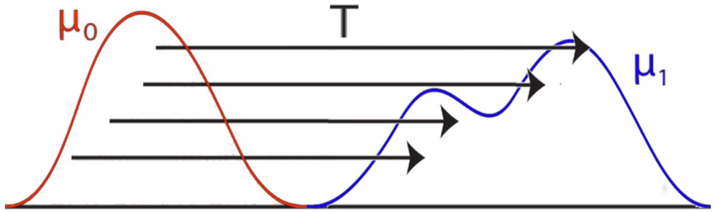


School bus Adversarial noise Ostrich

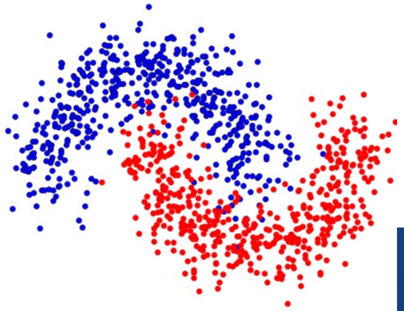
- Local Lipschitz robustness: $||f(x+\varepsilon) - f(x)|| \leq k.||\varepsilon||$

Rudolf Lipschitz
(source Wikipedia)

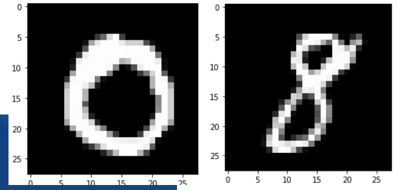
- Building 1-Lipschitz network



- Classification with Lipschitz Neural Networks



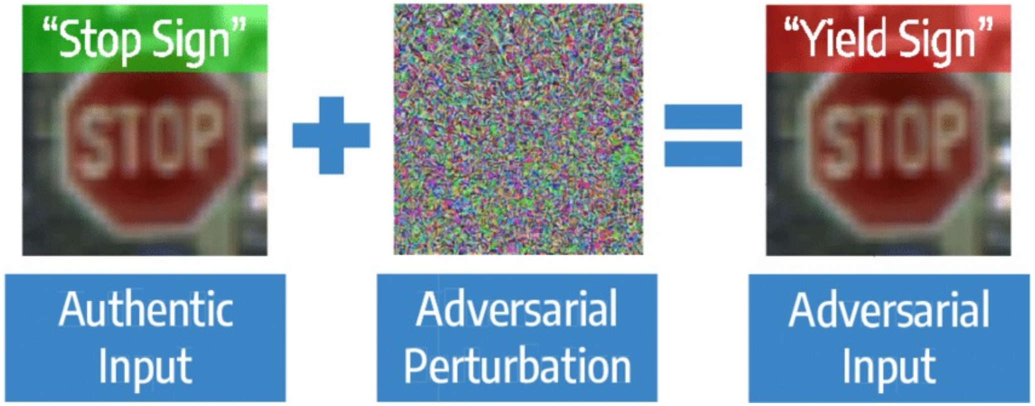
MNIST 0-8



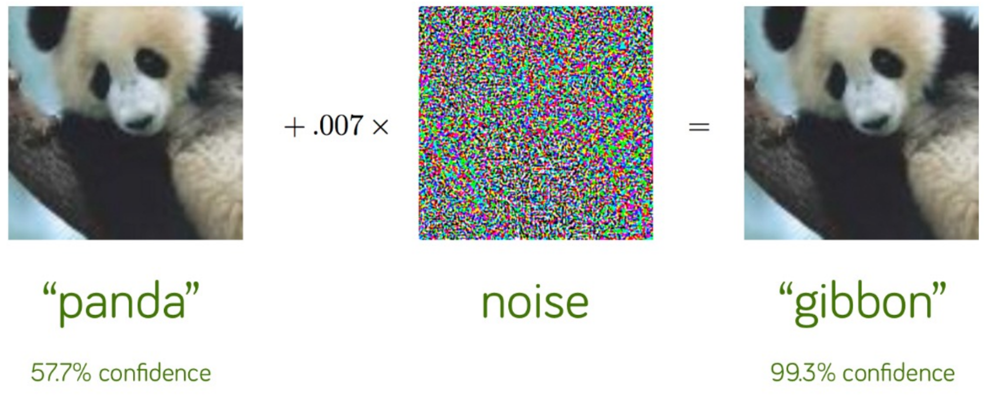
Blink left-right

Celeb-A (non)Moustache

- Adversarial attacks



ADVERSARIAL ATTACK



- Adversarial example : closest example with the opposite class

$$adv(f, \mathbf{x}) = \underset{\mathbf{z} \in \Omega | sign(f(\mathbf{z})) = -sign(f(\mathbf{x}))}{argmin} \| \mathbf{x} - \mathbf{z} \|$$

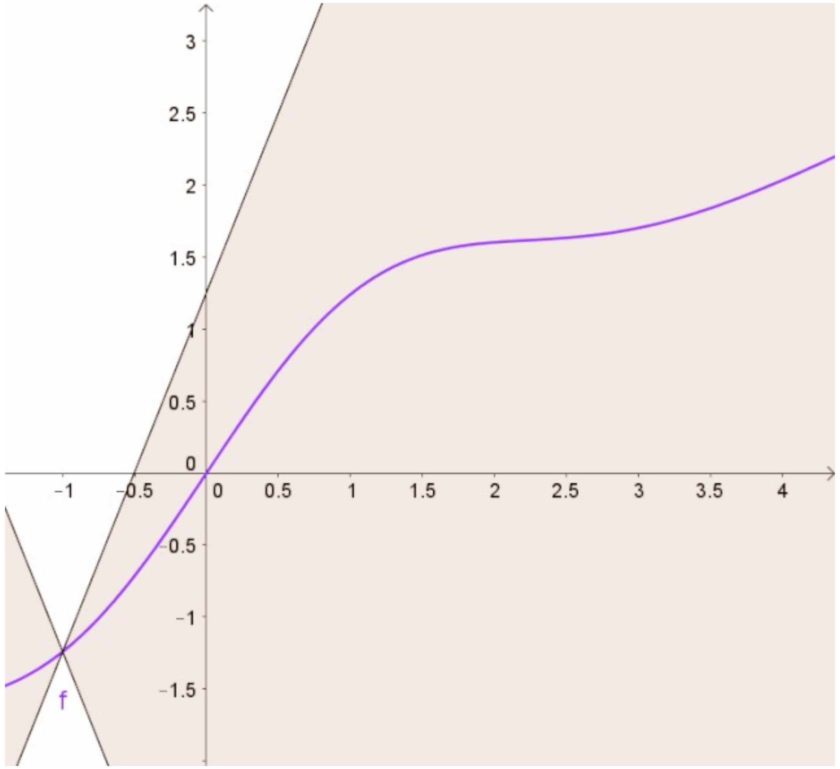
- Why neural networks are structurally weak to attack ?

LIPSCHITZ CONSTANT

- f: E->F is k-Lipschitz iif:

$$|| f(x_1) - f(x_2) || \leq k || x_1 - x_2 ||$$

- Lipschitz constant : smallest value of k
  - 1D case : k=max f'(x)
  - Intuition : how much the output of the fonction mary vary when I change the input



DEependable & Explainable Learning

LIPSCHITZ CONSTANT NEURAL NETWORK

- Very hard to evaluate accurately (np-hard)
- Multilayer perceptron

$$f(x) = \phi_k(W_k.(\phi_{k-1}(W_{k-1} \dots \phi_1(W_1.x)))$$

- Lipschitz constant upper-bound

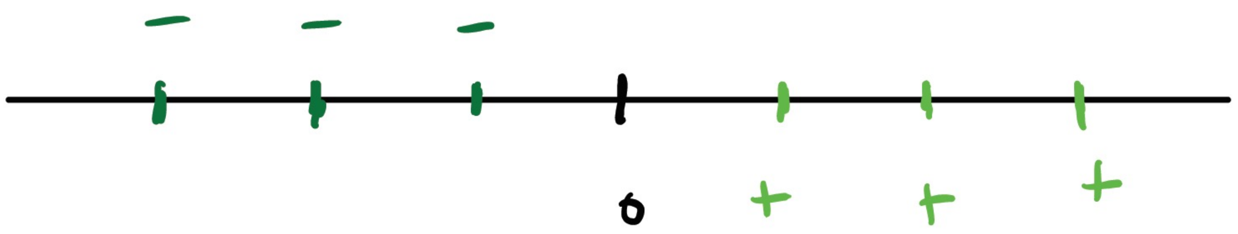
$$L(f) \leq L(\phi_k) * L(W_k) * L(\phi_{k-1}) * L(W_{k-1}) * \dots * L(\phi_1) * L(W_1.x)$$

- Activation function are usualy 1-Lischitz
- If all linear layer are 1-Lischitz, the network is 1-Lischitz (but the constant can be smaller than one)

WHY ARE NN WEAK TO ATTACK ?

- Maximizing cross entropy => increase the Lipschitz constant
- Lipschitz constant of NN grows exponentially
- High Lipschitz constant : small variation of inputs leads to high variation of the output -> principle of adversarial attack

- Illustration



- Building 1-Lipschitz neural networks

1-LIPSCHITZ NEURAL NETWORKS

- Principles : all the layers have to be 1-lipschitz
- Dense Layer with weights W

$$L(W) = ||W|| \leq ||W||_F \leq \max_{ij}(|W_{ij}|) * \sqrt{nm}$$

- Constraining Lipschitz constant
  - WGAN : weight clipping (last term of the equation)
  - Weight normalization with Frobenius norm $||W||_F$
  - Spectral normalization with spectral norm $||W||$
- Convolutional layers : same approach

1-LIPSCHITZ NEURAL NETWORKS

- Pooling layer : use L2 norm pooling
- Activation function :
    - Max min
    - Group sort
    - Full-sort

- BatchNormalization: Not lipschitz
- Dropout: Not Lipschitz

DEEL LIP LAYER LIBRARY

- Keras/tensflow and pythorch implementation
- Open source
- Keras extention
  - k-lischitz layers
  - activation functions
  - weight initializers
  - monitoring tools
- layer exportation (optimization for inference)

DEEL LIP LAYER LIBRARY

Standard neural network	Lipschitz neural network

Standard neural network

```python
from tf.keras.layers import (Conv2D,
    Dense, ReLU, BatchNormalization,
    MaxPool2D, Flatten)


model = tf.keras.Sequential(
    Conv2D(32, 3),
    BatchNormalization(),
    ReLU(),
    Conv2D(64, 3),
    MaxPool2D(),
    ReLU(),
    Flatten(),
    Dense(10)
)
```

Lipschitz neural network

```python
from tf.keras.layers import Flatten
from deel.lip.layers import (SpectralDense,
    SpectralConv2D, ScaledL2NormPooling2D)
from deel.lip.activations import GroupSort2

model = deel.lip.model.Sequential(
    SpectralConv2D(32, 3),
        # No batch normalization
    GroupSort2(),
    SpectralConv2D(64, 3),
    ScaledL2NormPooling2D(),
    GroupSort2(),
    Flatten(),
    SpectralDense(10)
)
```

DEEL-LIP LIBRARY

DEpendable & Explainable Learning

deel-lip

LIPSCHITZ KERAS LAYERS

Search docs

CONTENTS:

Installation

You can install `deel-lip` directly from pypi:

```
pip install deel-lip
```

In order to use `deel-lip`, you also need a valid tensorflow installation. `deel-lip` supports tensorflow 2.0 and tensorflow 2.1.

Cite this work

This library has been built to support the work presented in the paper *Achieving robustness in classification using optimaltransport with Hinge regularization*.

This work can be cited as:

```
@misc{2006.06520,
Author = {Mathieu Serrurier and Franck Mamalet and Alberto González-Sanz and Thibaut Boissin and Jean-Mi
Title = {Achieving robustness in classification using optimal transport with hinge regularization},
Year = {2020},
Eprint = {arXiv:2006.06520},
}
```

https://deel-lip.readthedocs.io/en/latest/

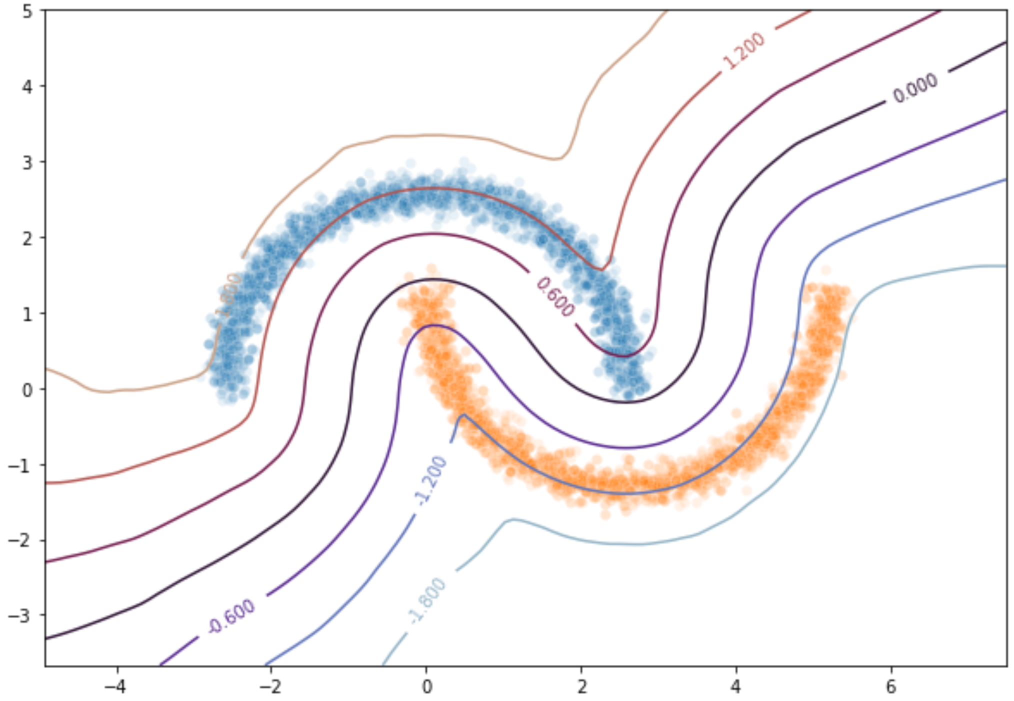
MODEL EXPORTATION

- This is done with `deel.model.Model` class
  - extends the original keras Model class
  - allows to call `model2 = model.vanilla_export()`

```
Model: "model"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_1 (InputLayer)         [(None, 28, 28, 1)]       0
_________________________________________________________________
flatten (Flatten)            (None, 784)               0
_________________________________________________________________
spectral_dense (SpectralDens
_________________________________________________________________
group_sort (GroupSort)       (None, 128)               0
_________________________________________________________________
spectral_dense_1 (SpectralDe
_________________________________________________________________
group_sort_1 (GroupSort)     (None, 64)                0
_________________________________________________________________
spectral_dense_2 (SpectralDe
_________________________________________________________________
group_sort_2 (GroupSort)     (None, 32)                0
_________________________________________________________________
normalized_dense (Normalized
=================================================================
Total params: 111,076
Trainable params: 110,849
Non-trainable params: 227
_________________________________________________________________
```

```
Model: "model_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_2 (InputLayer)         [(None, 28, 28, 1)]       0
_________________________________________________________________
flatten (Flatten)            (None, 784)               0
_________________________________________________________________
                      (Dense)    (None, 128)               100480
_________________________________________________________________
group_sort (GroupSort)       (None, 128)               0
_________________________________________________________________
                        (Dense)  (None, 64)                8256
_________________________________________________________________
group_sort_1 (GroupSort)     (None, 64)                0
_________________________________________________________________
                        (Dense)  (None, 32)                2080
_________________________________________________________________
group_sort_2 (GroupSort)     (None, 32)                0
_________________________________________________________________
                        (Dense)  (None, 1)                 33
=================================================================
Total params: 110,849
Trainable params: 110,849
Non-trainable params: 0
_________________________________________________________________
```

- Training 1-Lipshitz network

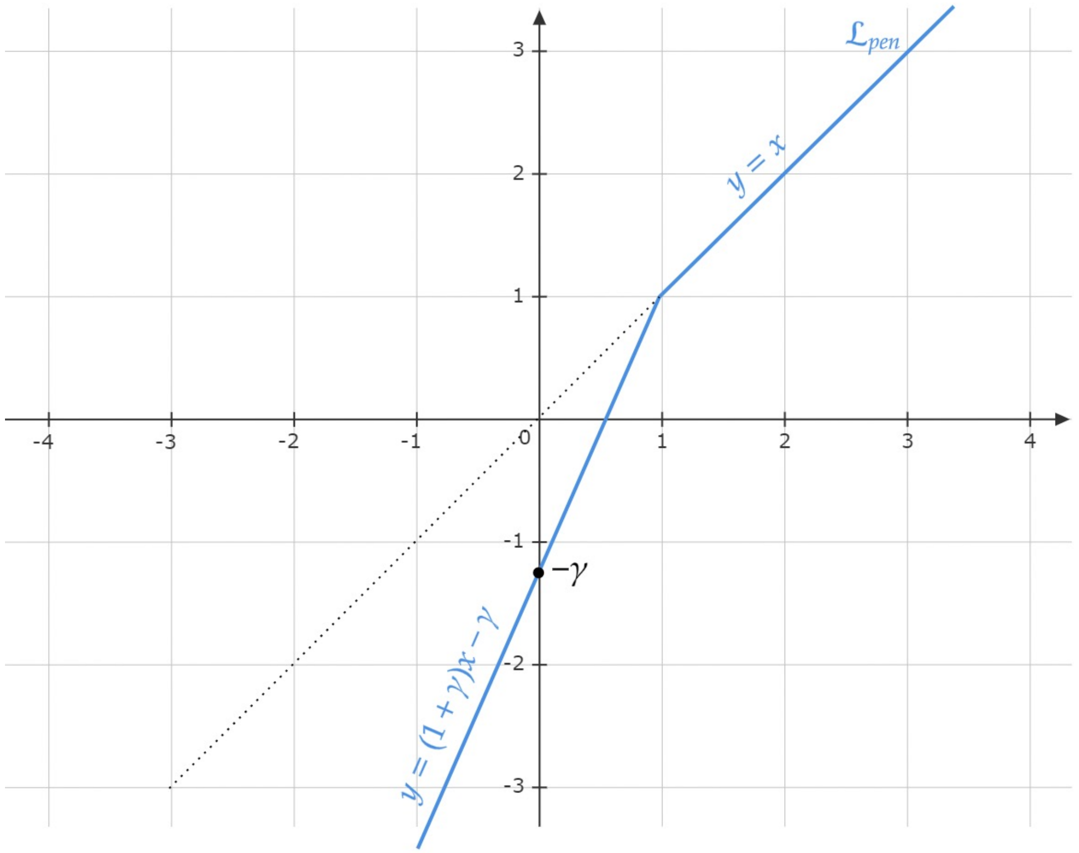


1-LIPSCHITZ TRAINING LOSS

- Problem : cross entropy not suitable for 1-Lipschitz networks
- Optimal transport loss :

$$\inf_{f \in Lip_1(\Omega)} \underset{\mathbf{x}\sim P_-}{\mathbb{E}}[f(\mathbf{x})] - \underset{\mathbf{x}\sim P_+}{\mathbb{E}}[f(\mathbf{x})] + \lambda \underset{\mathbf{x}}{\mathbb{E}}(1 - Yf(\mathbf{x}))_+$$

- Class = sign of f



PROPERTIES OF PROPOSED SOLUTION

- Link machine learning with optimal transportation
- Provable robustness

$$\| x - adv(\hat{f}, x) \| \geq |\hat{f}(x)|$$

- In practice

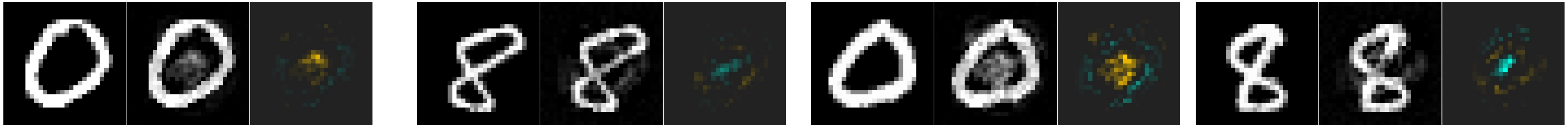
$$\| x - adv(\hat{f}, x) \| \simeq |\hat{f}(x)|$$

- Structurally Robust to attack
- Interpretable : Adversarial examples show explicitelly what you have to change to change class

- Experimentations

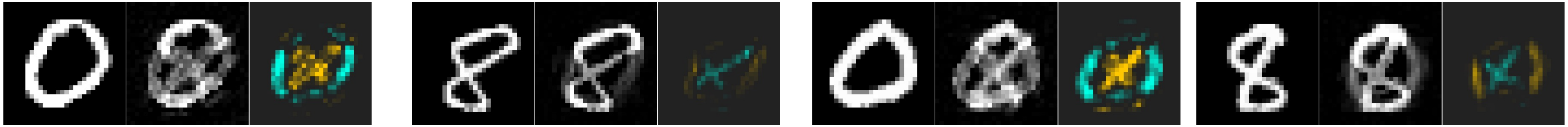
MNIST 0-8 ADVERSARAL ATTACKS

- Qualitative comparison of adversarial examples (50/50 score)



MLP with binary cross-entropy

1-Lipschitz MLP with binary cross-entropy



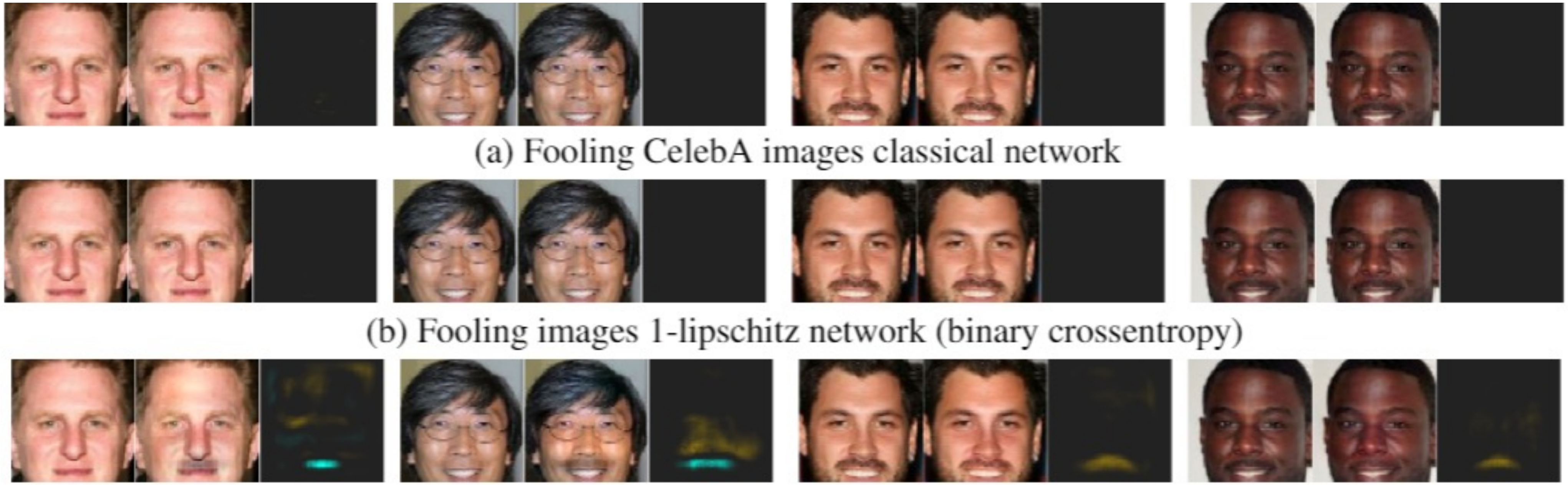
1-Lipschitz MLP with regularized OT

CELEB-A MOUSTACHE ADVERSARAL ATTACKS

- Qualitative comparison of adversarial examples (50/50 score)

Lipschitz VGG with **regularized OT**

Without moustache In-between with or without moustache



(a) Fooling CelebA images classical network

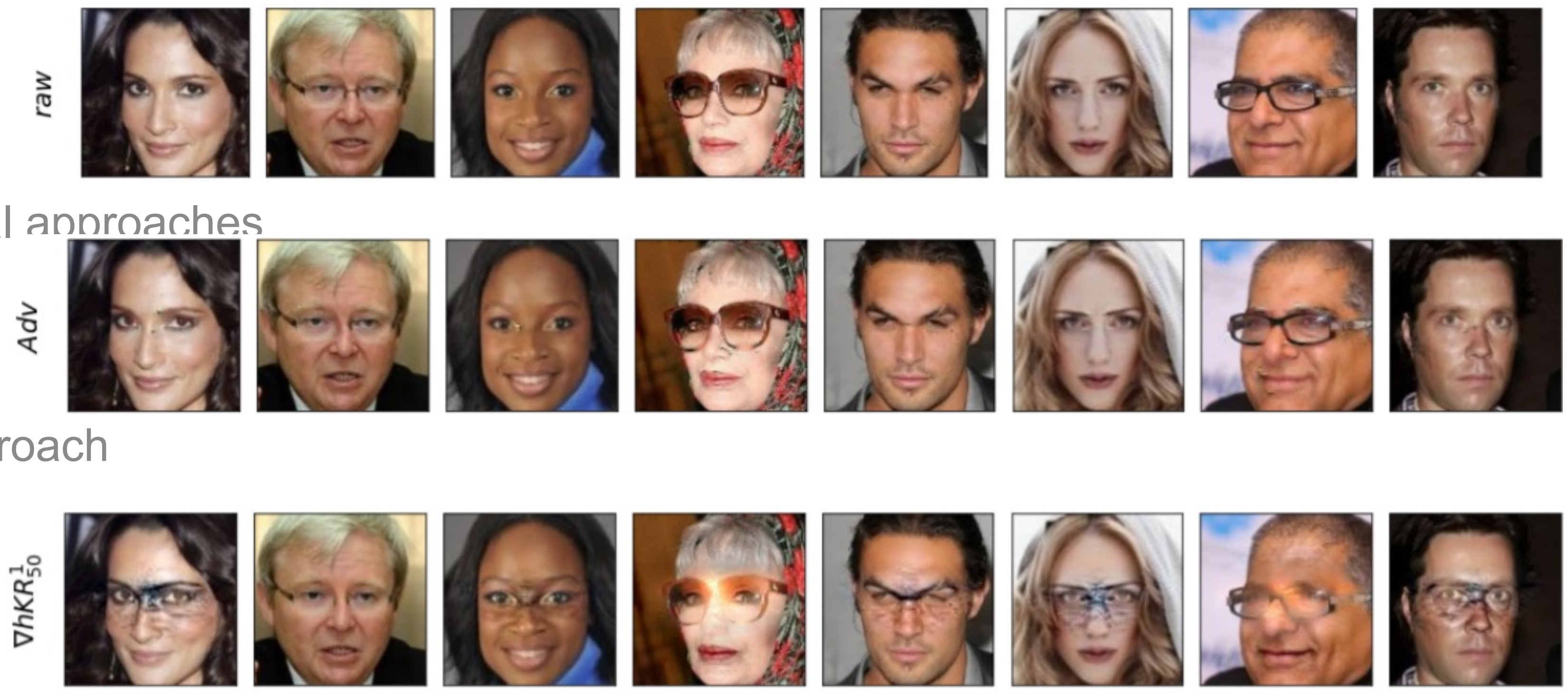
(b) Fooling images 1-lipschitz network (binary crossentropy)

CELEB-A GLASS ADVERSARAL ATTACKS

- Qualitative comparison of adversarial examples

Lipschitz VGG with **regularized OT**

- Raw
- Classical approaches
- Our approach



CONCLUSIONS

- Training 1-Lipschitz networks with optimal transport loss
  - New interpretation of classification problem
  - Improve robustness structurally
  - Meet certification requirement
  - Interpretable
- Deep lip : accessible and optimized library to train and use 1-Lipschitz networks
- Future works
  - Outlier detection
  - Semi-supervised/ agnostic learning